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TITLE- Low Altitude Orbits for Earth-Looking Experiments

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AUTHOR(S)- S. Shapiro

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ABSTRACT

[illegible] ...yzes the effect of orbital parameters and the instrument angle of view on the frequency of sighting of fixed targets on Earth.

For a particular instrument, the questions answered here are: "Given an orbit, what is the greatest interval between successive sightings of a given target?" and conversely, "for given sighting frequency, what ranges of altitude and inclination are permissible?"

General analytic results are obtained for low orbits, and graphical results are presented for representative systems: targets of latitudes 0° and 45°, half-angle of view 30° and 50°, circular orbits at altitudes 100 to 340 n.m., inclination 10° to 170°.

With a reasonably wide field of view, a suitable range of parameters for frequent observations of a given target will consist of a narrow band (or several narrow bands) of altitude over a wide range of inclinations. For a single target, inclinations approximately equal to the target latitude will give best results; for general coverage, a choice of inclination can be made to satisfy other constraints.

It is noted that the 50°, 270 nautical mile orbit often used for space station design lies on a repeating ground track and is very unfavorable for frequent coverage. At this inclination altitudes outside the range 270 ±25 n.m. are needed for coverage better than weekly, the exact value depending on the angle of view.

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FROM: S. Shapiro

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TECHNICAL MEMORANDUM

I. INTRODUCTION

The choice of an orbit for a scientific space mission depends in part on general considerations such as launch facilities, cost of station-keeping, radiation hazards, etc., and in part on factors closely related to the particular objectives of the mission. At the early stages of planning it is highly desirable that the implications of these various factors for the choice of orbit should be understood in a direct and straightforward way so as to facilitate an initial choice of approximate systems specifications. It is well known for example, that the radiation environment imposes clear restrictions on orbital altitude for manned flights; that high inclination is necessary for extensive earth-looking programs, and so on. Some features, however, require more extensive analysis to recognize their implications, and it is to one of these that this study is devoted.

A major feature of earth-looking programs is the requirement to make repeated observations of fixed targets. This may arise from the desire to follow the changing behavior of dynamic variables, or from the need to attain a degree of confidence in the data that a single observation cannot assure. The questions: How often, and under what conditions of illumination, will a given target be sighted from a spacecraft in a given orbit? do not appear to have received more than cursory general treatment.[1] The problem of optimal Earth coverage has recently been studied,[7] and there are, of course, computer programs[2] which will trace the ground track of a given spacecraft and hence provide the required information for each special case, but that sort of approach is suitable only when the special case has been selected. We seek a more general, though less precise, guide which will permit quick and direct comparisons between a variety of alternatives.

Illumination conditions are important in that certain types of observation can be made only when the lighting is suitable: Geological and cartographic applications demand illumination from particular Sun angles, and so on. Analysis of illumination conditions will be pursued separately.

The problem of sighting opportunity is most acute in low orbit; in very high orbits the entire face of the globe is in view at any time, and the apparent rotation of the Earth controls sightings.

The natural divide between the two classes of orbits is the altitude at which the field of view just contains the whole circle of the Earth ($\sim$1,430 n.m. for 45° half-angle of view). Numerical results for the entire range of altitudes up to this point would, however, become unwieldy. We can choose a more restricted but reasonable range of altitudes by considering only the class of orbits suitable for <u>manned</u> space flight.

Our discussion will be limited to low orbits because of the hazards of the radiation belts. If we adopt as a guide the dose limits(5)

Skin - 250 rads, Deep body - 25 rads;

we can find an upper bound on altitude derived from figures given by Hilberg.(6)

For a 100-day mission the allowable deep body dose would be 0.25 rad/day corresponding to altitude 330 n.m., inclination 60°, assuming total shielding (body and spacecraft) to be 5 gm/cm^2. The corresponding skin dose is about 1 rad/day (shielding 1 gm/cm^2), well below the limit. At lower inclinations, the dose will be a little higher so we may adopt 330 n.m. as a reasonable upper limit of altitude, though heavier shielding or shorter missions might permit higher altitudes. In the following sections, the model is developed and analyzed in mathematically convenient variables. The results and conclusions appear in sections V and VI.

II. <u>THE MODEL FOR THE TARGET ACQUISITION PROBLEM</u>

Let us rephrase the question and consider the first aspect of it: <u>How often will a given point fall within the field of view of a given satellite?</u>

Three factors determine the opportunities of observations:

1. The pattern of ground tracks
2. The position of the target
3. The field of view of the observing instrument.

We will seek to represent these factors by generalized parameters which may be applied without modification to all situations.

Ground Tracks

Assuming that the field of view is small in relation to the circular projection of the Earth (true for low orbits) it follows that sightings can occur only when the subpoint of the satellite is close to the target. In particular, it must be close to the parallel of latitude in which the target lies.

Therefore, our consideration of the pattern of ground tracks can be restricted to the pattern of crossing points of a given parallel of latitude. Furthermore, in steady state, the crossing points are repeated without limit at constant intervals, so that if we consider any pair of successive crossings, the pattern of subsequent crossings between that pair is identical to the pattern between any other such pair. Therefore, we can finally restrict our attention to the pattern of crossings on an interval (containing the target) between an arbitrary pair of successive crossings.

Definition: σ = the ratio of the arc of constant latitude between two successive crossings of the target latitude, to the perimeter of that circle of latitude. (Figure 2)

The value of σ completely determines the pattern of ground track crossings, except for the starting point of the pattern, which is arbitrary. (See Appendix B)

Position of the Target

It is intuitively clear (though a firm argument is presented in Appendix B) that since the ground track crossings repeat indefinitely at constant intervals, there will build up a pattern of points that is invariant with respect to any starting point whatever around the entire parallel of latitude. Therefore, the pattern of points in the neighborhood of any target at that latitude will be the same.

It follows that the longitude of the target does not enter into the analysis: Only the latitude is relevant.

The Field of View

For a viewing system fixed relative to the spacecraft the field of view will be a swath of constant width parallel to the ground track (Figures 1a and 2) Because we have restricted the region of interest to an arc σ of constant latitude, the part of the field of view that is relevant is the intersection of the swath with the parallel of latitude of the target.

Definition: f = the length of the arc of constant latitude lying within the field of view. (Figures 1a and 2)

f is measured as a proportion of the perimeter of the parallel of latitude, as is σ.

Times of Observation

An orbit will cross a given parallel of latitude at two points. Each crossing is potentially an opportunity for a sighting. There will, therefore, be two groups of sightings: Those occurring on the paths of increasing latitude and those on paths of decreasing latitude.

It is most convenient to consider each group separately, for the following reasons. Successive sightings belonging to one group will occur at similar conditions of illumination, because sightings can occur only when the target lies close to the orbital plane, and since the orbital plane scarcely changes in one day, the target returns to it at roughly 24-hour intervals. This requirement often forces us to consider sightings of one group only.

Furthermore, successive crossings of one group only - i.e., both on the rising arm or both on the falling arm - are exactly one period apart, whereas the time between successive crossings in either direction is a function of latitude, eccentricity, and argument of perigee. Even when the complete sequence of observations is required it is easier to combine the two separate sequences than to analyze the single series of time-varying intervals. (See Appendix D)

III. ANALYSIS OF FLYOVER POINTS

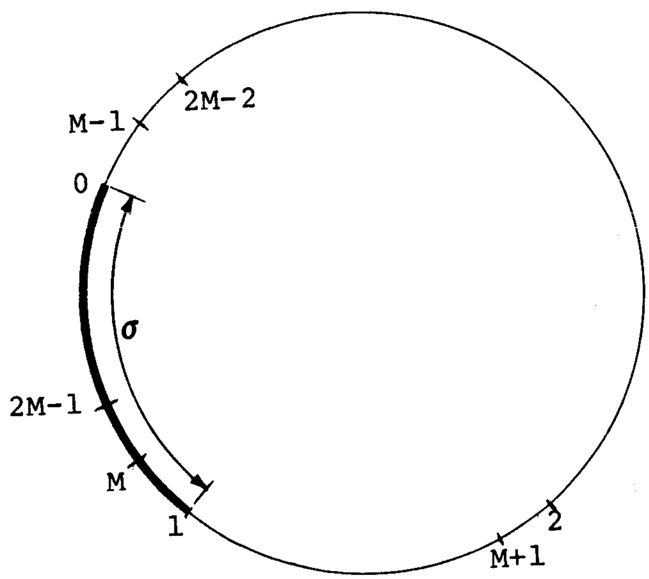
The analysis will depend only upon the generalized parameters, σ, f.

The entire process of interest takes place on an arc of length σ, where the zero'th and first crossings occur at the end-points of the arc. The second crossing will fall outside of the arc, but the next to fall on it will be the M'th, about one day later, where

$$(1 + \sigma) \geq M\sigma \geq 1 \quad . \tag{1}$$

(Recall that the perimeter of a circle of constant latitude has unit length.)

M is the number of revolutions required to circumnavigate the globe.



We will introduce a new term:

Definition: a <u>cycle</u> = N revolutions = the number of revolutions between successive crossings of an arc of length σ.

It is evident that N does not have a unique value, for if one crossing occurs near the left end-point of σ, the next must be to its right; on the other hand for a crossing near the right end-point, the next must fall to its left.

In general, N may take one of two values for a given satellite, viz, the two integers between which lies the value $\frac{1}{\sigma}$. Thus, M=1 + integral part of $1/\sigma$, and N = M or M - 1. In Fig. A, the labels denote the number of cycles that have elapsed up to that crossing. The fifth and tenth cycles have values $N < 1/\sigma$; others have $N < 1/\sigma$.

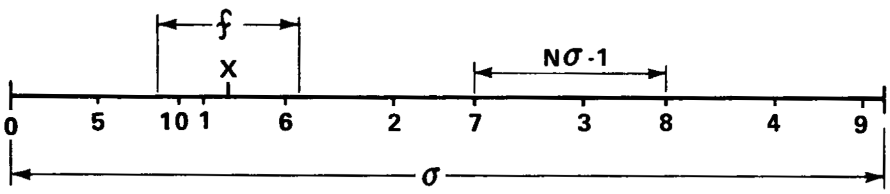


Fig. A

Suppose a target is at X. If X falls within the interval f corresponding to some crossing, a sighting will occur.

Note that the relation between f and its crossing point depends upon the observing system. For nadir photography the crossing point is at the mid-point of f (See Fig. 1). For oblique photography it might not be within f at all. Nevertheless the relative positions of successive f-intervals follow precisely the positions of successive crossing points, though displaced by some constant amount.

Thus, we may, without loss of generality, consider only the symmetric case shown in Fig. A. Other cases will give the same sequence of sightings, but starting at a different cycle. Since the starting point is in any case arbitrary, this does not constitute a relevant distinction.

For the symmetric case, then, X will be sighted at all crossings which fall within an arc $\pm$ f/2 from X. This sequence of crossings does not follow an obvious pattern, has no periodicity, and is very sensitive to the values of

σ and f. This fact sheds a new light on the original question: "How often do sightings occur?" for it is apparent that "how often" implies a regularity that does not in fact obtain.

We must, therefore, pose a new problem: <u>What is the maximum number of cycles between successive sightings of a target at a given latitude from a given satellite?</u> The answer to this problem will be of the worst-case type, since the maximum may occur relatively infrequently.

The method of solution is based upon the construction of Fig. A. Suppose a sighting occurs at cycle 1. If we find a position for f such that the maximum number of subsequent crossings are excluded from f we shall have found the maximum number of cycles intervening between sightings for the target at the mid-point of f. This result will hold equally for any target point at that latitude. If f is sufficiently large, both 1 and 2 must be included, then the maximum number of subsequent cycles will be excluded, and so on.

We do not in fact have to make the implied determination of the positions of f explicity. It is evident that the situation is controlled by the two parameters σ, f, and the (σ-f) plane will be divided into separate regions each corresponding to a constant value of the maximum number of cycles between sightings.

Denote this maximum value: C.

We prove in Appendix C that the constant-C regions are all triangles, and devise a simple construction for them. The resulting partition of (σ,f) space appears in Fig. 3.

<u>Explanation of Figure 3</u>

The pattern of Fig. 3 and the value of the maximum number of cycles between sightings reflect two major properties of the system: (a) the relative magnitudes of the field-of-view (f) and the space between tracks (σ), and (b) the shift in longitude of the ground track after one circumnavigation of the globe ($N\sigma$-1).

A larger field-of-view for a given σ will cover relatively more ground, and give more frequent coverage thus, C decreases as f increases. If f is sufficiently large, sightings can be made at least once each cycle. Figure 2 shows that this will be the case if successive swaths overlap, i.e., $f \geq \sigma$. This is the uppermost region in Fig. 3.

Using a similar concept for smaller fields-of-view, when swaths on the second cycle just fill in the gaps left on the first cycle, sightings can be made at least every 2 days. This can occur for $f \geq \frac{1}{2}\sigma$, and in Fig. 3 the regions C=2 lie above, and touch, the line $f = \frac{1}{2}\sigma$. In general, C=p touches from above the line $f = \frac{1}{p}\sigma$.

But the size of f is not the only criterion. If the tracks are phased so that the swaths on successive cycles repeat and do not "cover the gaps", then coverage is not efficient and a point above the line $f = \frac{1}{2}\sigma$ might still require more than 2 cycles for complete coverage of that latitude. This reflects property (b) above.

The extreme case of this is the precisely repeating track where overlap is exact and the gaps are never filled in. This corresponds to the vertical lines in Fig. 3 occurring at rational values of $\sigma = \frac{m}{n}$, implying precise repetition in n revolutions, which is m cycles,(roughly m days). Some areas will be sighted at every repetition, others will never be covered. In this case the sighting opportunities depend upon the longitude of the target. For an operating point, on a vertical line which terminates in a region C=p, targets in the covered area will be seen every p days, targets outside will never be seen. Thus, these lines appear in Fig. 3 as the limit as $C \to \infty$.

It must be emphasized that Fig. 3 gives the most pessimistic point of view. A high value of C may indicate that the tracks are so phased that overlap is considerable, and it takes many cycles before the gaps between swaths are covered. On the other hand it implies also that a target covered in one cycle is likely to be covered in many successive ones because of the overlap. Thus, for nearly repeating tracks there will be a period of daily sightings followed by a long period of no sightings; it is the latter that is given by Fig. 3.

IV. RESULTS IN TERMS OF THE PHYSICAL PARAMETERS

For any practical application of these results the generalized parameters σ,f must be related to the physical variables altitude, inclination, latitude, etc. In this section explicit expressions are given for σ and f in terms of altitude, inclination, target latitude, and angle-of-view. Corresponding inverse expressions are also given for altitude and inclination as functions of σ and f, and the results are shown graphically for some representative cases.

Physical Explanation of σ

As defined in Section II above, σ is the length of arc of constant latitude between two successive ground tracks, measured relative to the perimeter of the circle of latitude. The displacement between successive tracks is due to (a) the Earth's rotation, and (b) the rotation of the orbital plane. (Atmospheric drag, higher gravitational harmonics, and other perturbations, are neglected.) If the period of revolution is P days, the Earth makes P rotations during one revolution. If the orbit rotates at a rate $\frac{360}{2\pi}\dot{\Omega}$ deg./day, then the relative rotation between the Earth and the orbital plane is $\sigma = P[1 - \frac{\dot{\Omega}}{2\pi}]$ rotations per satellite revolution.

We have

$$P = \frac{2\pi a^{3/2}}{\mu^{1/2}} \text{ days/rev.} \tag{2}$$

and

$$\dot{\Omega} = - \frac{JR^2 \cos i \cdot \mu^{1/2}}{a^{7/2}(1-e^2)^2} \tag{3}$$

$$\therefore \sigma = \left\{ \frac{2\pi a^{3/2}}{\mu^{1/2}} + \frac{JR^2 \cos i}{a^2(1-e^2)^2} \right\} \text{ rotations per satellite revolution} \tag{4}$$

where

R = radius of Earth

μ== geogravitational constant

i = orbital inclination

Ω = longitude of the ascending mode

J = oblateness parameter

e = eccentricity

For low circular orbits a good approximation is

$$\sigma = \left[\frac{2\pi R^{3/2}}{\mu^{1/2}} + J \cos i\right] + \left[\frac{3\pi R^{3/2}}{\mu^{1/2}} - 2J \cos i\right]\frac{h}{R} \tag{5}$$

(h=altitude)

neglecting $(\frac{h}{R})^2$ and higher powers, so σ is roughly linear with altitude, and weakly cosinusoidal with inclination.

Physical Explanation of f

If the instrument maintains a fixed inclination to the local vertical, and has a constant angle of view, the limits of the field of view will sweep out circles on the Earth, parallel to the satellite's ground track which is a great circle. Figure 1 shows this for a nadir-pointing instrument.

The two points at which these circles intersect the parallel of latitude of the target, are the end-points of the arc f. In Appendix A expressions are derived for f applying to an instrument at altitude h, with half-angle θ, pointing in a direction ψ from the vertical, normal to the ground track.

We find

$$f = \frac{1}{2\pi}\left\{\sin^{-1}\frac{\sin\phi \cos i - W_1}{\cos\phi \sin i} - \sin^{-1}\frac{\sin\phi \cos i - W_2}{\cos\phi \sin i}\right\} \tag{6}$$

where

$$W_{1,2} = \sin(\psi \pm \theta)\cos(\psi \pm \theta)\left[1 + \frac{h}{R} - \left(1 - \frac{h}{R}(2 + \frac{h}{R})\tan^2(\psi \pm \theta)\right)^{1/2}\right] \tag{7}$$

(A slightly different form applies at latitudes near the value of inclination. See Appendix A)

The gross characteristics of this expression may be grasped by considering the simplest special case; that is, nadir-pointing observations (ψ=o) of equatorial targets ($\emptyset$=o) from a high-inclination orbit (sin i >> W) at low altitude $\left|\left(\frac{h}{R}\right)^2 << 1\right|$ with half-angle not exceptionally wide ($\theta < 45°$).

Then a binomial expansion of the square root in (7) gives

$$W_{1,2} = \pm \frac{h}{R} \tan \theta \left[1 + \frac{h}{R} \tan^2 \theta\right]$$

neglecting powers of $\frac{h}{R}$ greater than the square. An idea of the degree of approximation involved is obtained by noting that for h < 340 n.m., θ < 45, we have $\frac{h}{R} \tan^2 \theta < .1$. Neglecting this quantity, we have together with (5), the approximate relation:

$$f \simeq \frac{\tan \theta}{\pi \sin i} \cdot \frac{h}{R} \tag{8}$$

Contours in the (σ-f) plane

Rewrite (5) as $\sigma \simeq (A + J \cos i) + (\frac{3}{2} A - 2J \cos i) \frac{h}{R}$ (9)

where

$$A = 2\pi R^{3/2}/\mu^{1/2}, \qquad \text{and} \qquad e = o .$$

Numerically,

$$\sigma \simeq 10^{-2} \left\{5.87 + .16 \cos i + (8.80 - .32 \cos i) \frac{h}{R}\right\}$$

(4), (6) and (7) or their approximations (8), (9) will provide the values of the parameters f (field of view) and σ (displacement between ground tracks) for any orbiting system. Figure 3 then gives the corresponding value of C, the maximum number of cycles (or, roughly, days) between successive observations of a target at the given latitude.

f and σ for a variety of system parameter values are shown in Figures 4 through 7. A discussion of these curves motivated by physical considerations is presented below, but mathematically they can be understood by rearranging (8) and (9) into

$$f = \frac{\tan\theta}{\pi \sin i} \; \frac{\sigma - (A + J\cos i)}{(\frac{3}{2} A - 2J\cos i)} \tag{10}$$

and

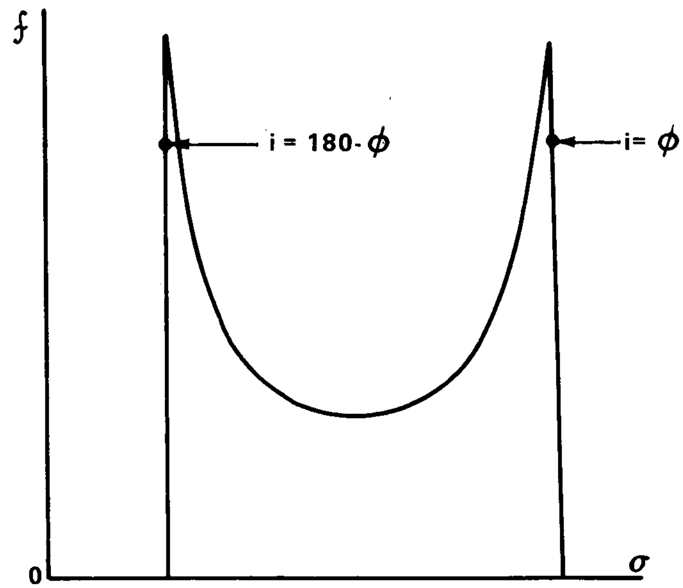
$$f = \frac{\tan\theta}{\pi} \frac{h}{R} \left[1 - \left(\frac{\sigma - A(1 + \frac{3}{2}\frac{h}{R})}{J(1 - 2\frac{h}{R})} \right)^2 \right]^{-\frac{1}{2}} \tag{11}$$

Thus for fixed i, f is linear in σ and proportional to $\tan\theta$. For fixed h/R, f has a minimum at $\sigma = A(1 + \frac{3}{2}\frac{h}{R})$, corresponding to $i \to 90$ (cf. (8)). Equation (11) indicates a steep rise with asymptotes at $\sigma - A(1 + \frac{3}{2}\frac{h}{R}) \pm J(1 - 2\frac{h}{R}) = 0$, i.e., $i=0°$, $180°$. This is correct for equatorial points, but for other latitudes the asymptotes occur near $i = \phi$.

Explanation of Figures 4 through 7

Greater values of f correspond to higher altitudes and lower inclinations. The actual width of the field-of-view depends on altitude and, of course, camera angle, but the latitudinal section, f, (see Fig. 2) will increase for lower inclinations. This is reflected in Figures 4 through 7.

There is some dependence on latitude: the angle between the ground track and the parallel of latitude decreases as latitude increases (Fig. 2) until at $\phi = i$, it is zero. The value of f will increase correspondingly. Compare Figures 4 and 6: In going from $\phi=0°$ to $\phi=45°$ points of given inclination remain at the same value of σ, but the value of f increases by an amount depending on i. The complete contours are not shown in figures 4 through 7. As i drops to the value of latitude ϕ (or rises to $(180-\phi)$) f increases rapidly (Fig. 8) to a peak, then falls abruptly to zero. The complete contour is indicated below.



The omission from the charts of the extremities is for the sake of clarity, and the resulting loss of information is not serious.

Expressions in Terms of Altitude and Inclination

In practice, results obtained in terms of the generalized parameters σ,f, (for example, Fig. 3) must be transformed into corresponding values of h and i. This may be done graphically using Figs. 3 through 7, or by solving equations (4), (6), (7) for h and i. A solution in closed form does not appear to be possible, but a numerical solution can be computed. For the most important case of nadir sighting (ψ=o) from circular orbit (e=o), these equations can be transformed into

$$\cos i = \left[\sigma - \frac{2\pi R^{3/2}\left(1+\frac{h}{R}\right)^{3/2}}{\mu^{1/2}}\right] \frac{\left(1+\frac{h}{R}\right)^{2}}{J} \qquad (4)'$$

$$W = \tan \pi f \left[\sin^2 i(1-\cos^2 \phi \sin^2 \pi f) - \sin^2\phi\right]^{1/2} \qquad (6)'$$

$$\frac{h}{R} = (1-w^2)^{1/2} + W \cot \theta - 1$$

Substituting (6)' into (7)' for W, and (7)' into (4)' for h, (4)' takes the form

$$\cos i = F(i,\sigma,f,\theta,\phi)$$

which can be solved for i, given the other parameters. (6)', (7)' then give the corresponding h.

V. Results in Altitude and Inclination

Figures 9-12 show the graphical results in altitude and inclination. As in Figure 3, the value of C corresponding to a given orbit corresponds to the maximum interval between sightings of a target. The typical interval may be less.

The contours of constant σ are the same on each chart and run roughly horizontally, rising about 100 nautical miles with inclination from 0° to 180°. The rational values correspond to repeating ground tracks, with $\sigma = 1/16$ and $\sigma = 1/15$ being very unfavorable for general coverage. Higher order values can be favorable, m day coverage being achieved first for $\sigma = m/n$ at the lower inclinations. With wider field of view, the m-day bands extend inwards along the σ contour toward high inclination. Note that the orbit 50°, 270 nautical miles, which is often discussed for space stations, lies on $\sigma = 1/15$ and is not favorable for Earth viewing.

Figure 9, corresponding to relatively narrow angle (30° half angle) instruments and equatorial targets, is the most severe. For 50° inclination, repetitive viewing at intervals of 5 days or less is available only above 300 n.m., and in very narrow bands at 235 and 205 n.m.

Figure 10, for 50° half angle of view (100° full angle), shows opportunities for 3-day coverage in three bands. For 50° inclination orbits, these are 160-175 n.m., 195-233 n.m., and 300-318 n.m. The first, at $\sigma = 3/47$, is just open at high inclinations.

Coverage is much more easily obtained for high target latitudes. Figures 11 and 12 cover the case of site latitude 45° Note that even here, $\sigma = 1/15$ and $1/16$ correspond to very unfavorable orbits.

VI. Conclusions

The analysis leads to the following conclusions:

(1) Most frequent coverage is given by the lowest inclination orbit which will cover all target latitudes.

(2) Wide angles of view, or at least substantial off nadir pointing is required. For 30° half-angle of view, and high inclination, no coverage more frequent than 5 days can be obtained below 300 n.m. Fifty degree half-angle allows 3-day coverage in several altitude bands.

(3) Repeating ground tracks, corresponding to $\sigma = 1/15$ and $1/16$ are very unfavorable, and must be avoided. $\sigma = 1/15$ passes through the 50°, 270 n.m. orbit commonly used as a design point for space stations. Assuming that the orbit is not precisely repeating, there will be periods of very frequent viewing of a given target, followed by long intervals when the target is not seen at all. For frequent (weekly) sighting of arbitrary targets, altitudes about 20 n.m. lower or higher than 270 n.m. are required.

1011-SS-cp

S. Shapiro

Attachments
- References
- Figures 1 through 12
- Appendices A through D
- Notations

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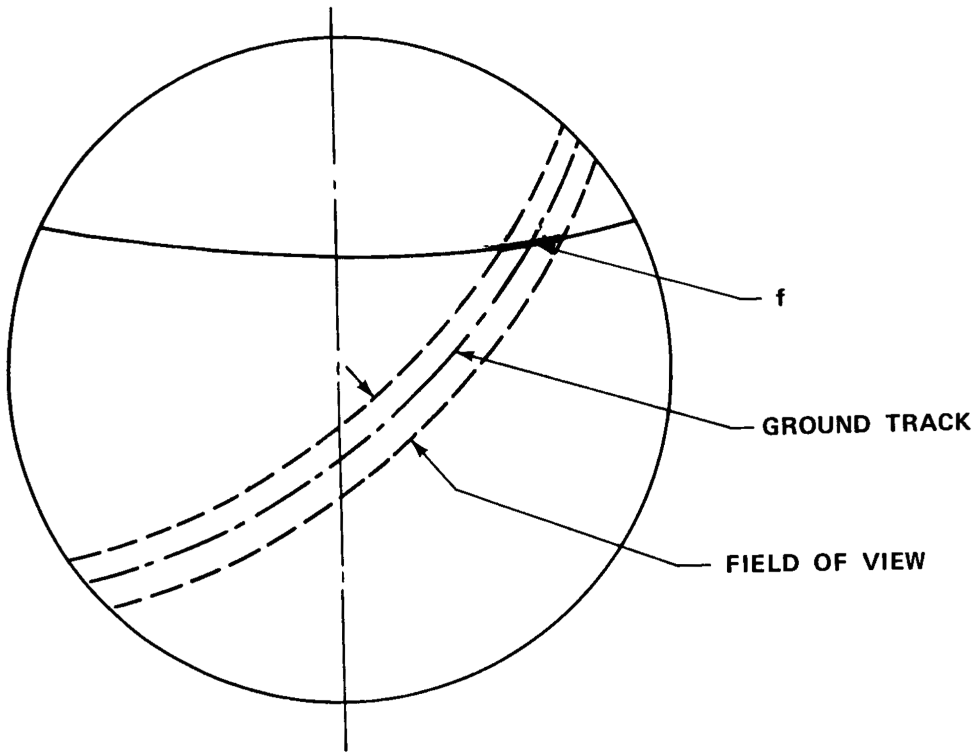


FIGURE 1a

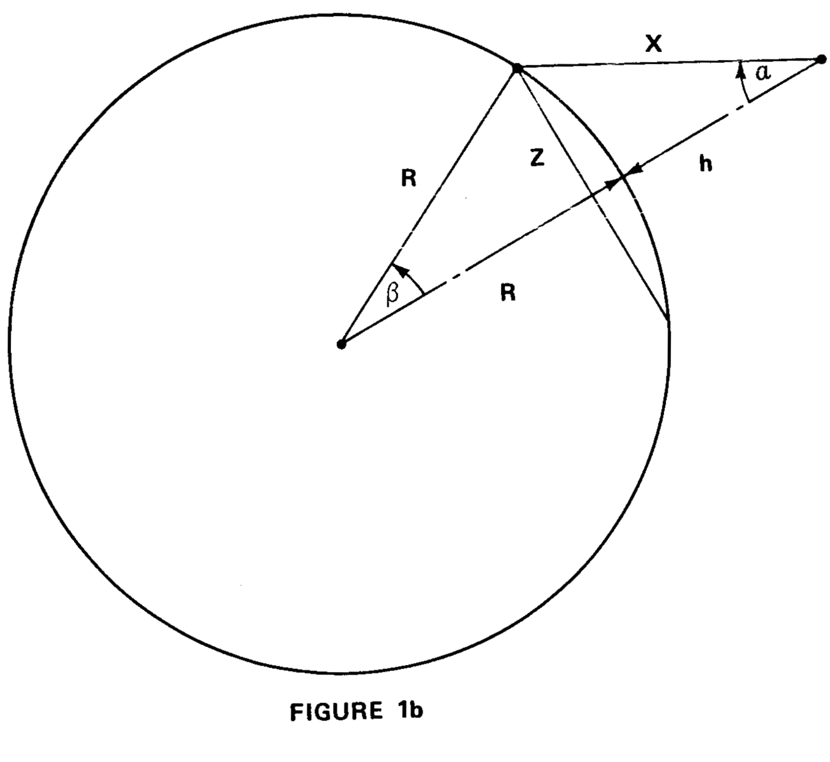


FIGURE 1b

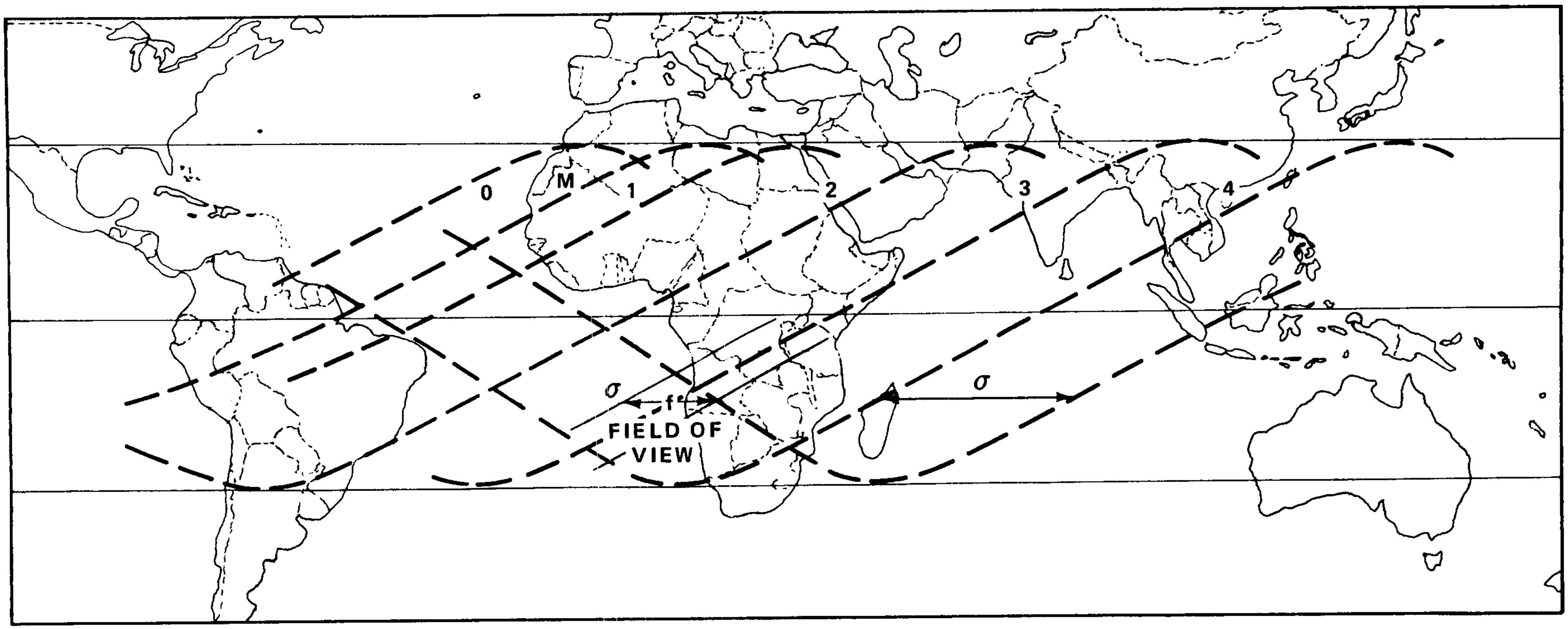


FIGURE 2 - PATTERN OF GROUND TRACKS ON SUCCESSIVE REVOLUTIONS STARTING AT TRACK 0. CIRCUMNAVIGATION IS COMPLETED AFTER M REVOLUTIONS

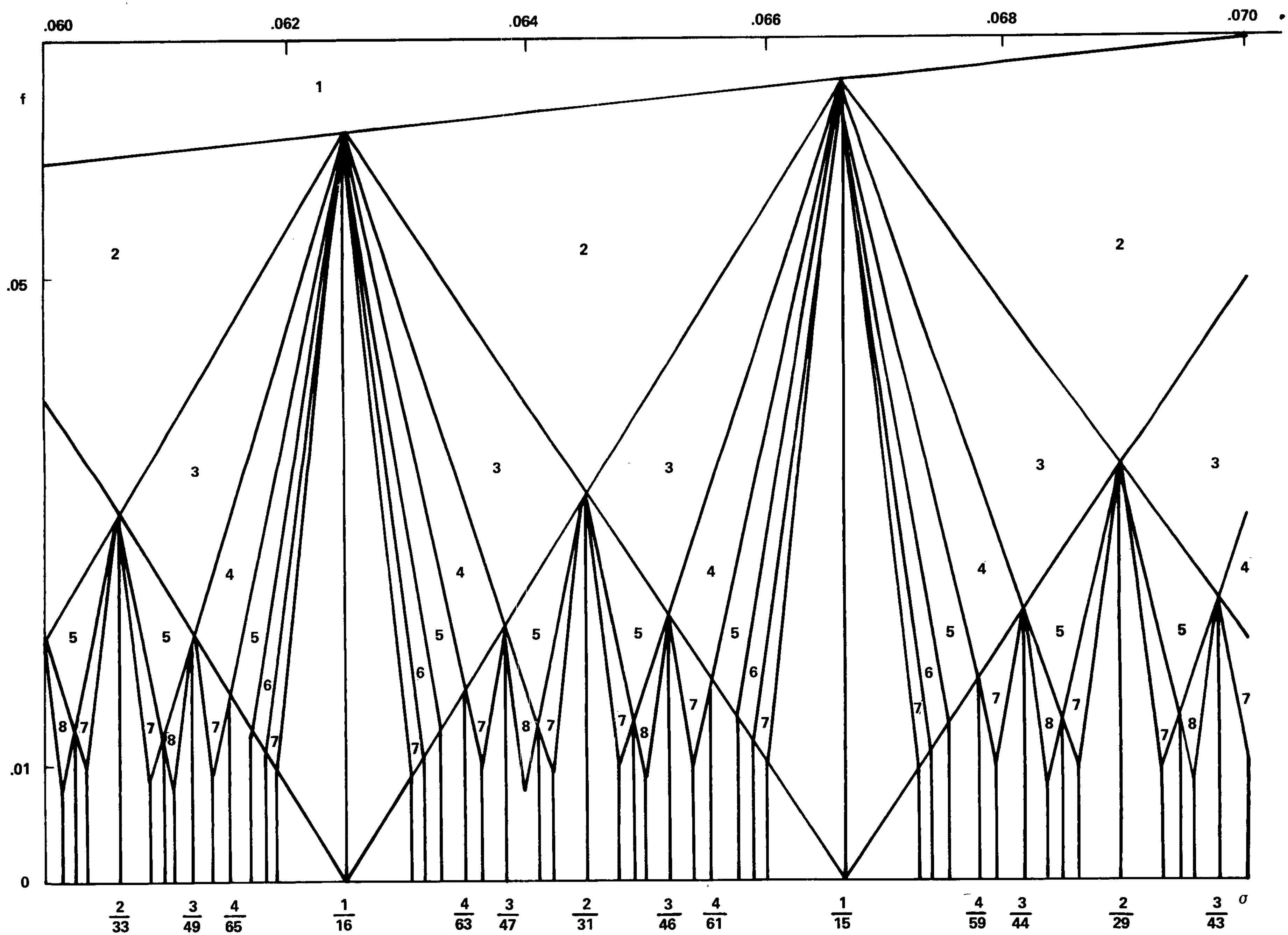


FIGURE 3 - FIELD OF VIEW VS. DISPLACEMENT BETWEEN GROUND TRACKS. (UNIT = PERIMETER OF PARALLEL OF LATITUDE) SHOWING MAXIMUM NO. OF CYCLES BETWEEN SIGHTINGS OF A FIXED TARGET

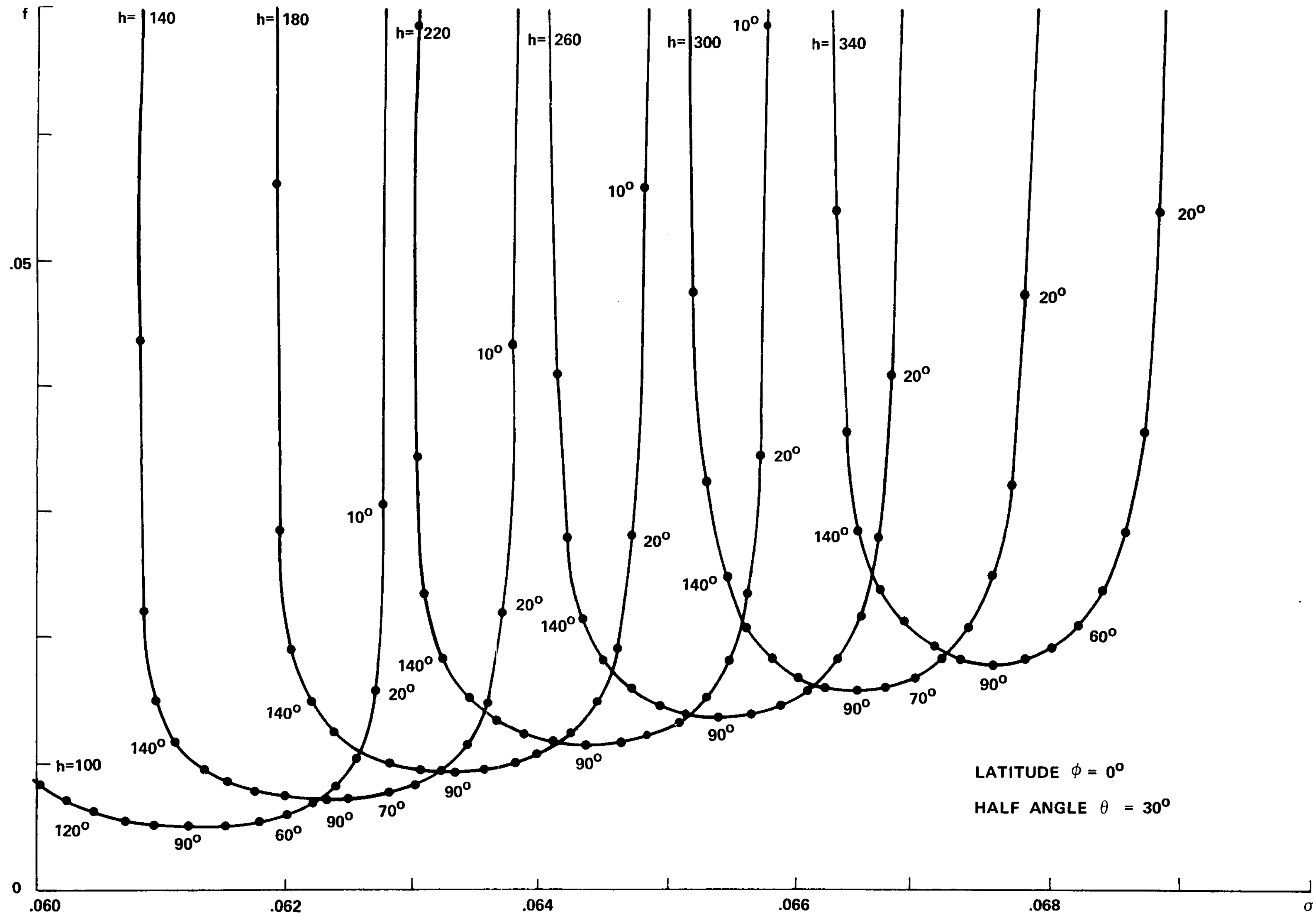


FIGURE 4 - LINES OF CONSTANT ALTITUDE IN THE (σ, f) PLANE. POINTS MARKED AT 10° INTERVALS OF INCLINATION

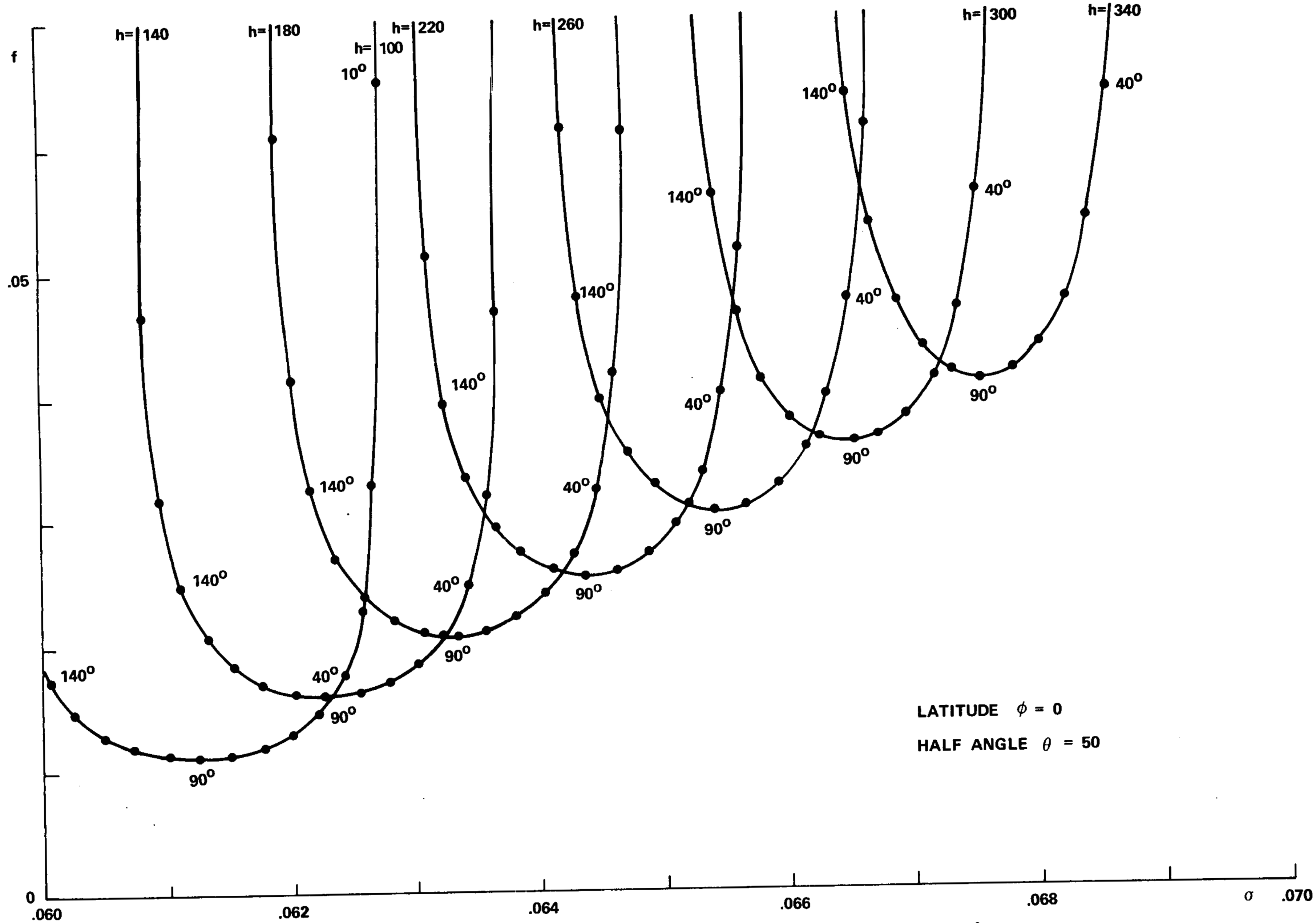


FIGURE 5 - LINES OF CONSTANT ALTITUDE IN THE (σ, f) PLANE. POINTS MARKED AT 10° INTERVALS OF INCLINATION

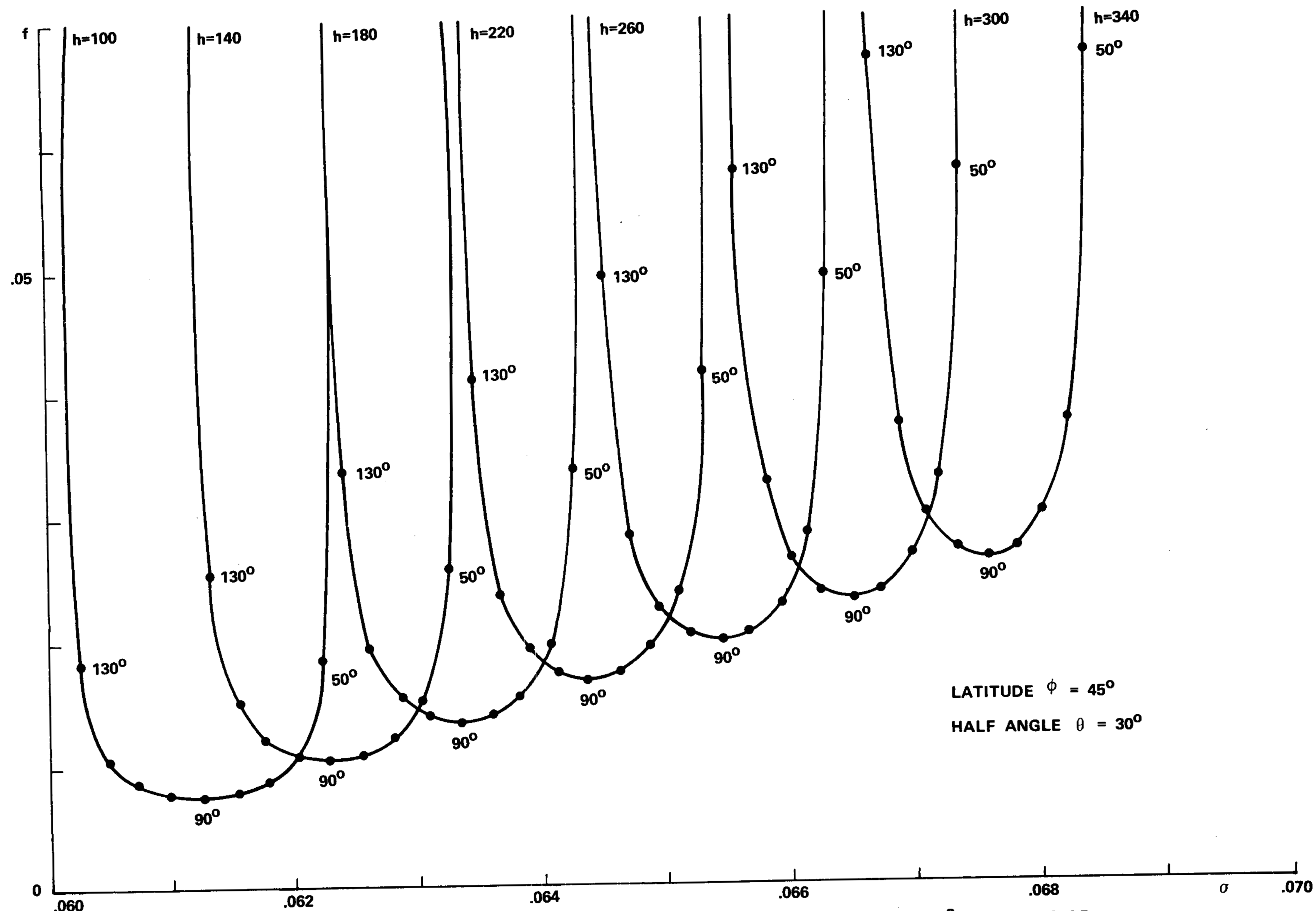


FIGURE 6 - LINES OF CONSTANT ALTITUDE IN THE (σ, f) PLANE. POINTS MARKED AT 10° INTERVALS OF INCLINATION

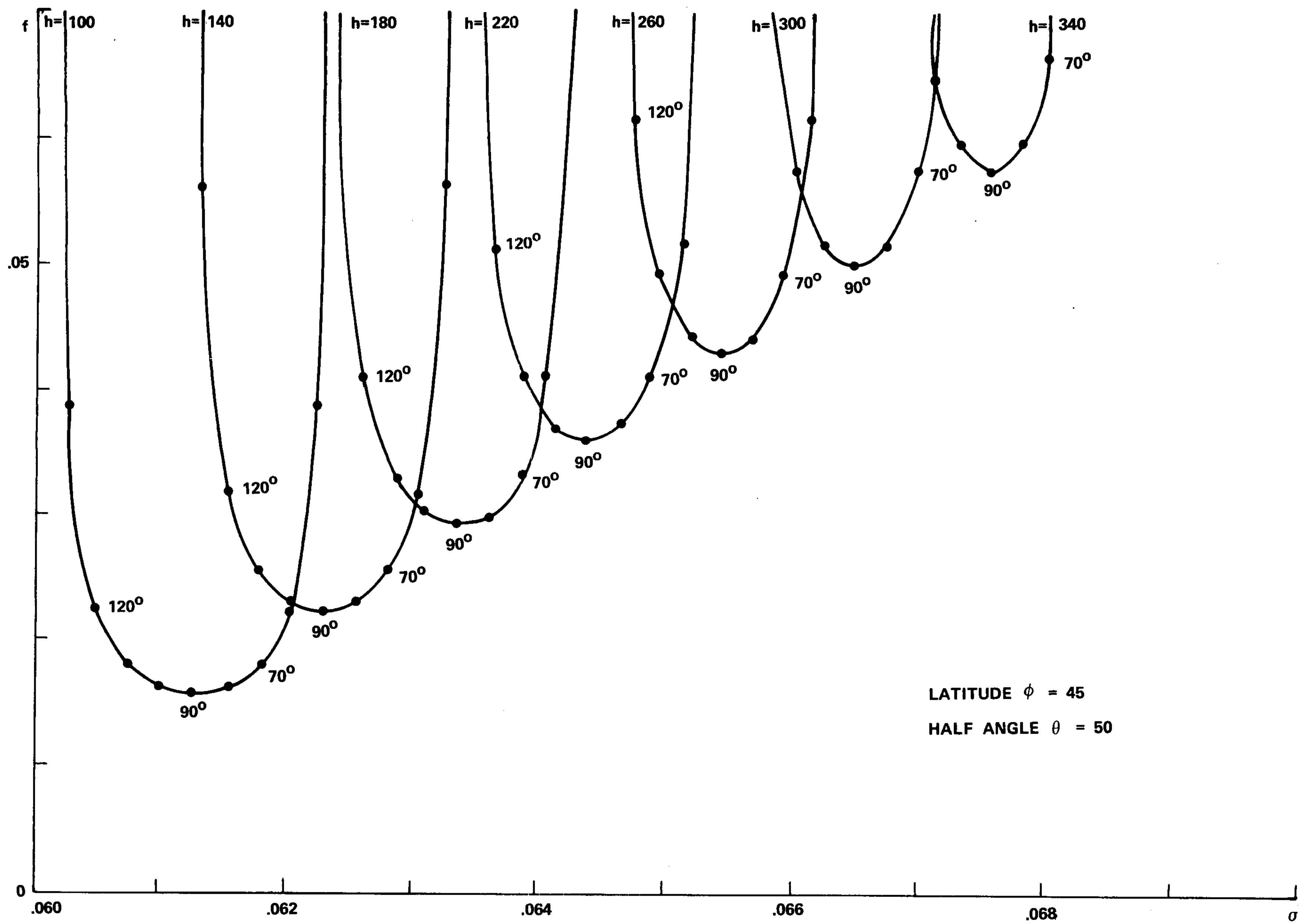


FIGURE 7 - LINES OF CONSTANT ALTITUDE IN THE (σ, f) PLANE. POINTS MARKED AT 10° INTERVALS OF INCLINATION

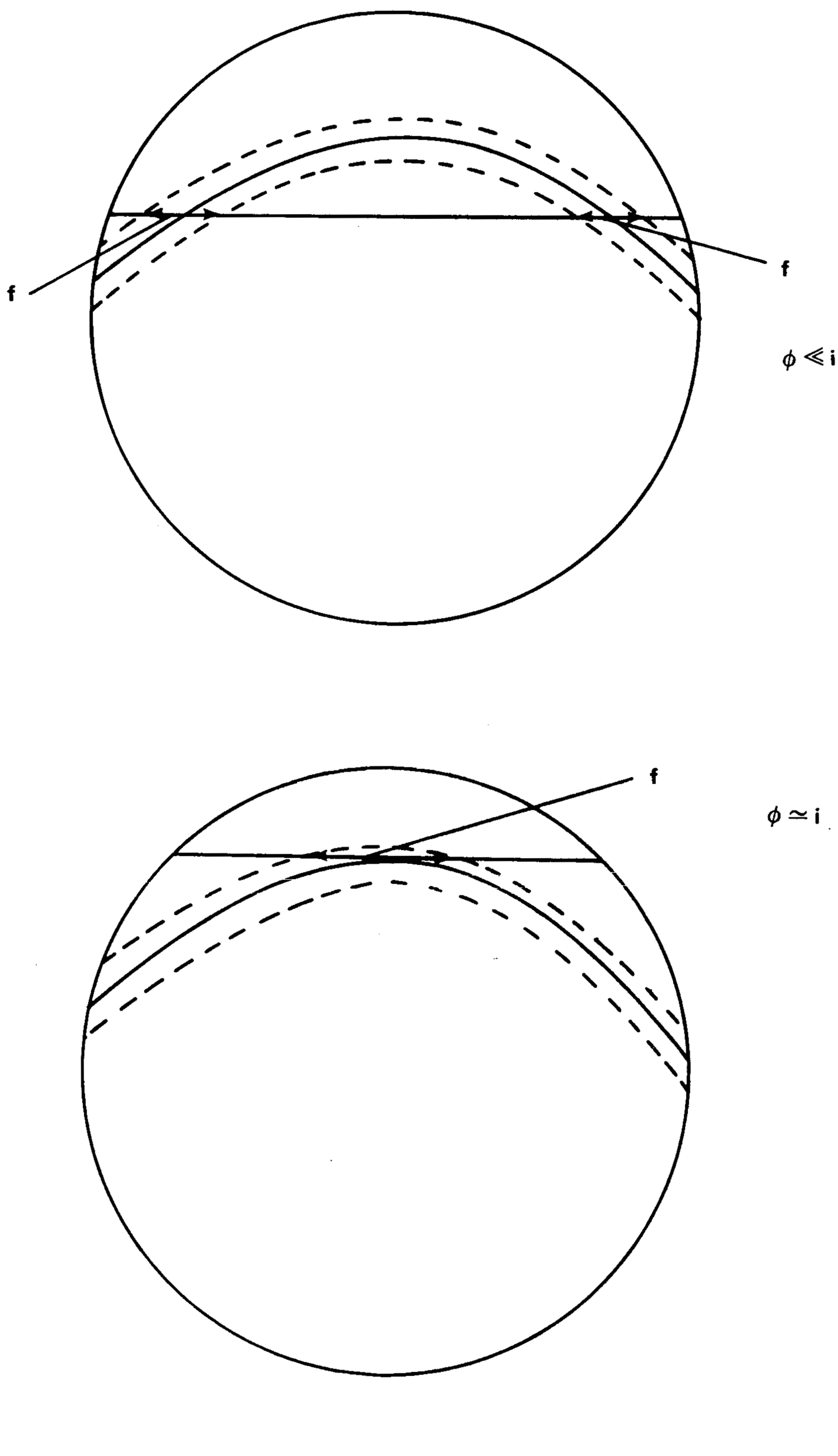


FIGURE 8 - INTERSECTION OF FIELD OF VIEW WITH PARALLEL OF LATITUDE

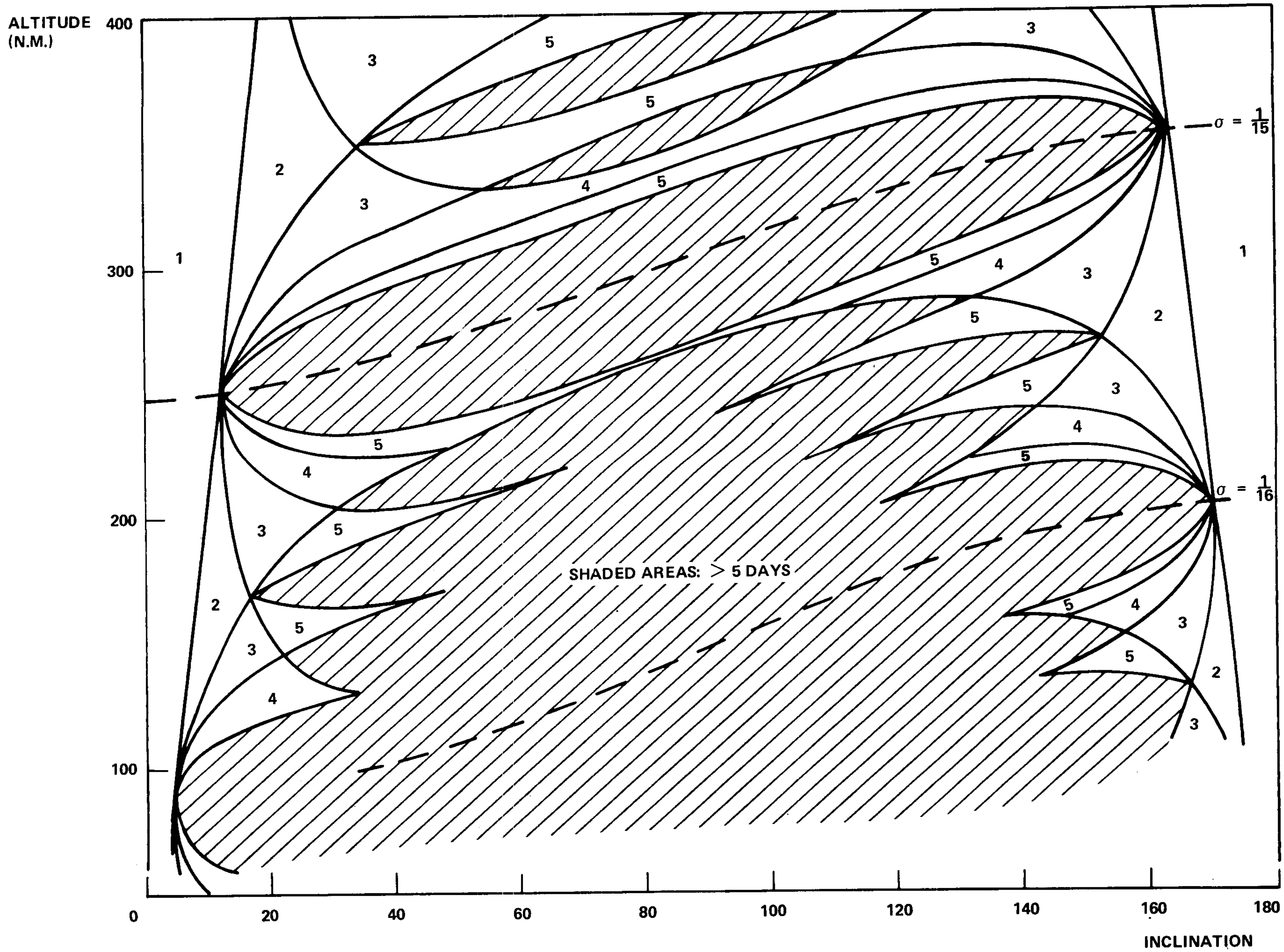


FIGURE 9 - MAXIMUM NO. OF DAYS BETWEEN SIGHTINGS OF A GIVEN TARGET AT LATITUDE 0°
HALF-ANGLE OF VIEW 30°

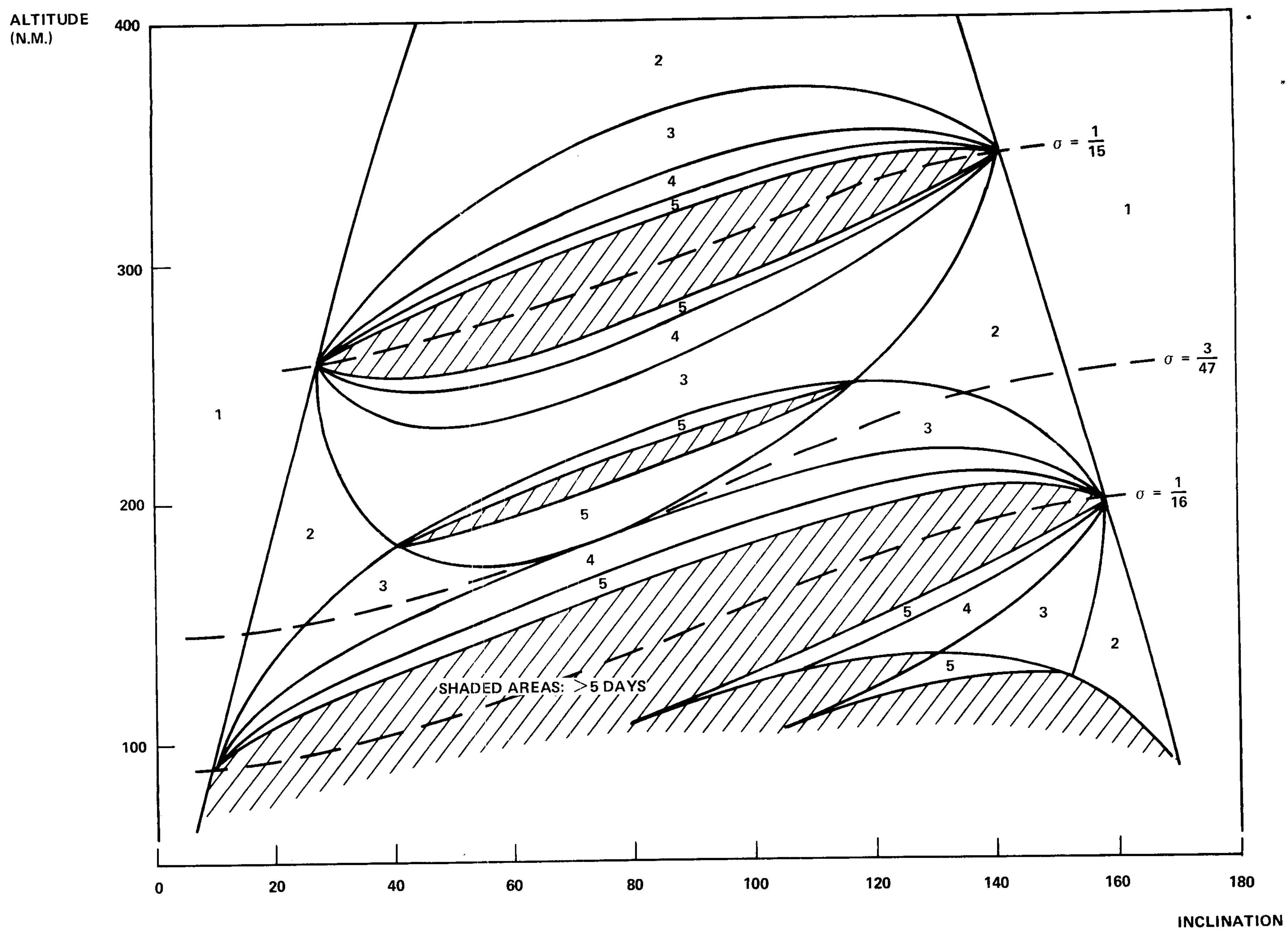


FIGURE 10 - MAXIMUM NO. OF DAYS BETWEEN SIGHTINGS OF A GIVEN TARGET AT LATITUDE 0°
HALF-ANGLE OF VIEW 50°

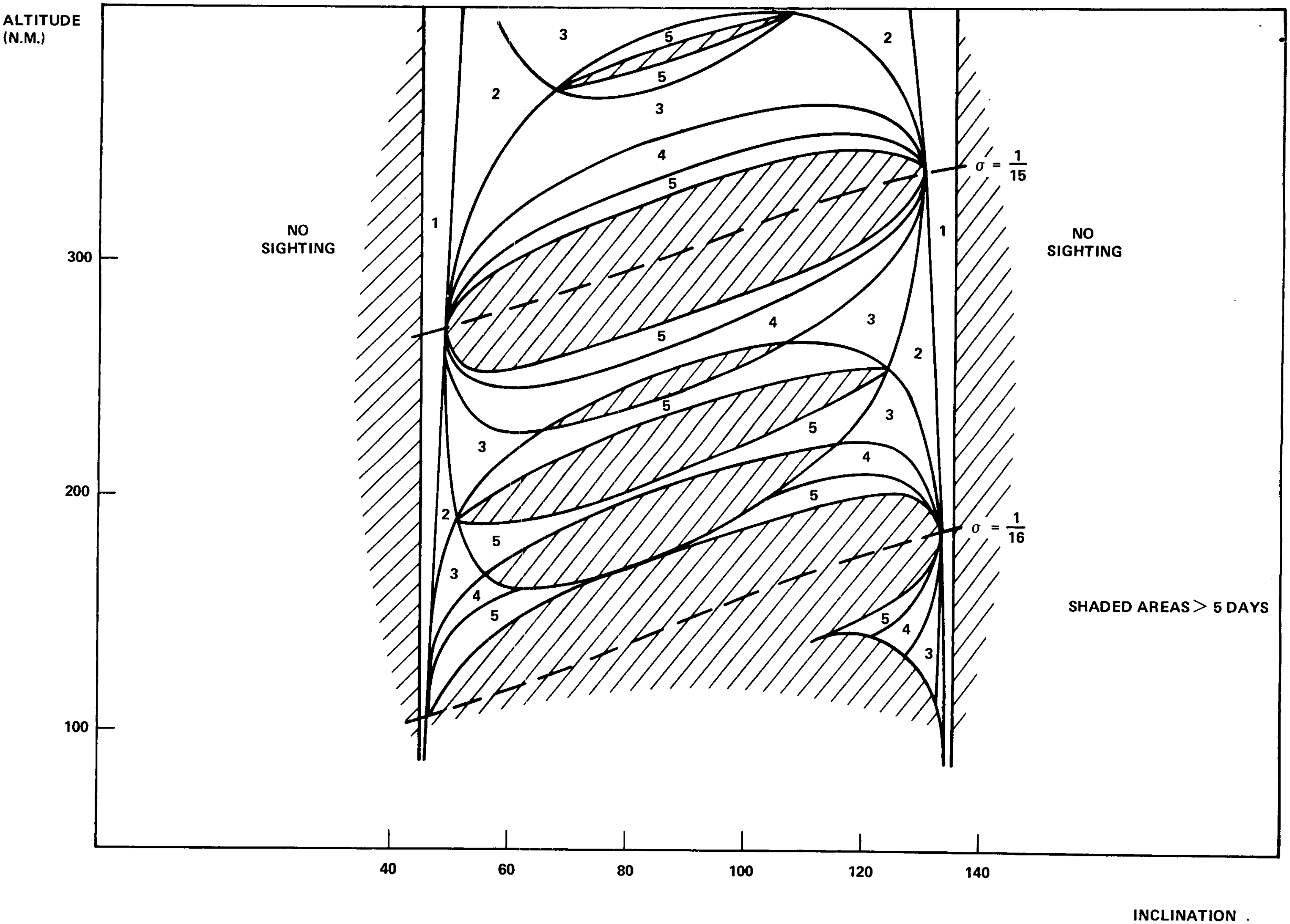


FIGURE 11 - MAXIMUM NO. OF DAYS BETWEEN SIGHTINGS OF A GIVEN TARGET AT LATITUDE 45°
HALF-ANGLE OF VIEW 30°

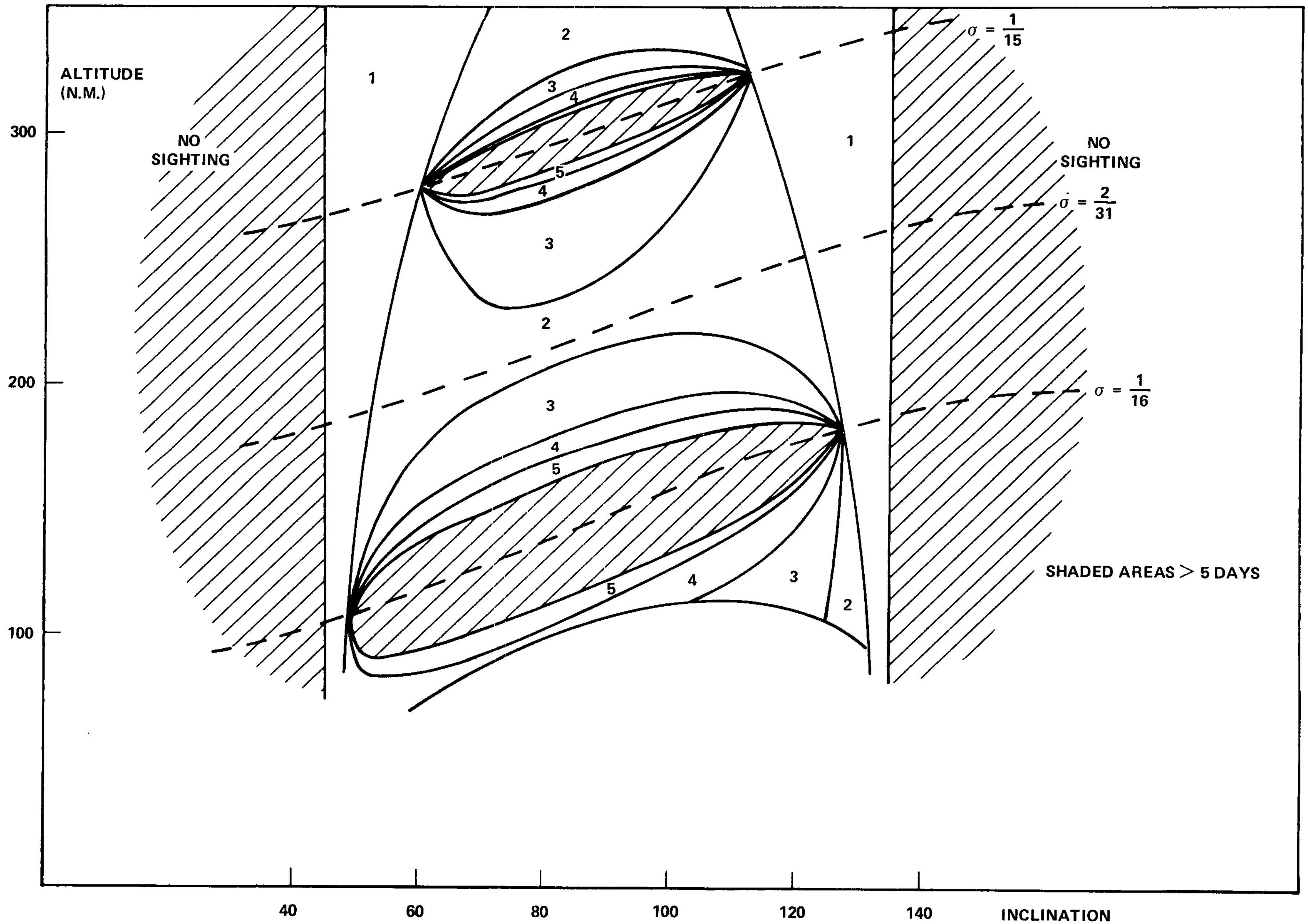


FIGURE 12 - MAXIMUM NO. OF DAYS BETWEEN SIGHTINGS OF A GIVEN TARGET AT LATITUDE 45°
HALF-ANGLE OF VIEW 50°

APPENDIX A

Evaluation of f

f is the intersection between the field of view from orbit and a parallel of latitude. Consider any line of sight from the satellite to the Earth. If this line is at a constant angle to the local vertical through the satellite, its endpoint will describe a circle on the Earth, whose plane is at a constant distance Z from the orbital plane. We will determine the distance Z.

Consider a plane through the Earth's center, containing the satellite. Construct in this plane a line from the satellite to the Earth. Z will be the distance from the diameter containing the satellite, to the endpoint of this line.

In Fig. 1b.

$$\frac{R}{\sin\alpha} = \frac{X}{\sin\beta}$$

and

$$X\cos\alpha + R\cos\beta = R + h\ .$$

Eliminating β we have

$$X\cos\alpha + R\left[1 - \frac{X^2}{R^2}\sin^2\alpha\right]^{1/2} = R + h$$

hence

$$\frac{X}{R} = \cos\alpha\left\{1 + \frac{h}{R} - \left(1 - \tan^2\alpha\cdot\frac{h}{R}\cdot\left(2 + \frac{h}{R}\right)\right)^{1/2}\right\} \qquad \text{(A.1)}$$

and

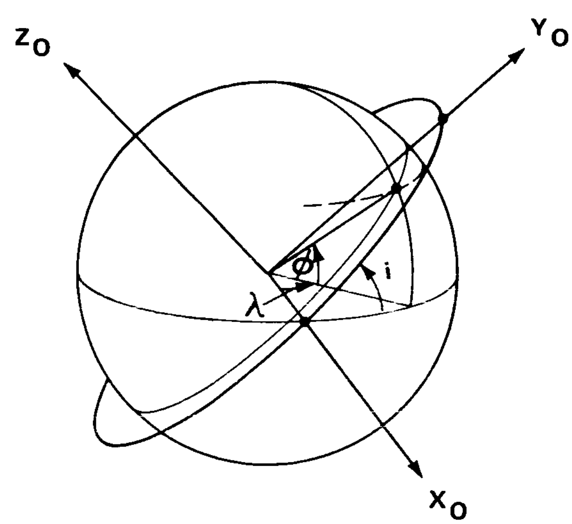
$$Z = X\sin\alpha = R\sin\alpha\cos\alpha\left\{1 + \frac{h}{R} - \left(1 - \frac{h}{R}\tan^2\alpha\left(2 + \frac{h}{R}\right)\right)^{1/2}\right\} \qquad \text{(A.2)}$$

If the camera points at an angle ψ to the vertical, and the half-angle is θ, then the extreme lines of sight correspond to

$$\alpha = (\psi \pm \theta) \quad .$$

In order to determine f we need the points of intersection of the circles just determined, with the parallel of latitude $\emptyset$.

Consider a system of orbital plane coordinates X_o, Y_o, Z_o with X_o along the line of nodes, and a latitude-longitude system $(R,\emptyset,\lambda)$ with longitude λ measured from the line of nodes.



The transformation $(R,\emptyset,\lambda)\rightarrow(X_o,Y_o,Z_o)$ is effected by

$$\begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} R \cos \emptyset \cos \lambda \\ R \cos \emptyset \sin \lambda \\ R \sin \emptyset \end{bmatrix} \tag{A.3}$$

Then the circles described by (A.2) intersect latitude ϕ at λ given by

$$R \sin\alpha\cos\alpha\left\{1+\frac{h}{R}-\left[1-\frac{h}{R}\left(2+\frac{h}{R}\right)\tan^2\alpha\right]^{1/2}\right\} = R(\sin\phi\cos i-\cos\phi\sin i\sin\lambda)$$

$$\sin\lambda = \frac{-\sin\alpha\cos\alpha\left[1+\frac{h}{R}-\left(1-\frac{h}{R}\left(2+\frac{h}{R}\right)\tan^2\alpha\right)^{1/2}\right]+\sin\phi\cos i}{\cos\phi\cdot\sin i} \tag{A.4}$$

Solutions will exist if

$$\sin(i-\phi) > -\sin\alpha\cos\alpha\left[1+\frac{h}{R}-\left(1-\frac{h}{R}\left(2+\frac{h}{R}\right)\tan^2\alpha\right)^{1/2}\right]$$

Three possible situations arise:

(i) (A.4) holds for both values $\alpha = \psi \pm \theta$, in which case we compute the two corresponding solutions λ_1, λ_2, and set

$$f = \frac{1}{2\pi}[\lambda_1 - \lambda_2] \tag{A.5}$$

(ii) (A.4) holds only for one value $\alpha = \psi + \theta$. Corresponding solutions are λ_1, $(\pi - \lambda_1)$, and

$$f = \frac{1}{2\pi}[\pi - 2\lambda_1] \tag{A.6}$$

(iii) (A.4) does not hold for either value of α, in which case no points at that latitude ever fall within the field of view.

Some simplification arises from nadir pointing observations, $\psi = 0$. In this case $\alpha = \pm\theta$ and (A.4) becomes

$$\sin\lambda = \frac{\sin\phi\cos i \pm \sin\theta\cos\theta\left[1+\frac{h}{R}-\left(1-\frac{h}{R}\left(2+\frac{h}{R}\right)\tan^2\theta\right)^{1/2}\right]}{\cos\phi\sin i}$$

For the special, but important case of equatorial targets, we have $\phi = o$, and

$$f \simeq \frac{\sin\theta\cos\theta\left[1 + \frac{h}{R} - \left(1 - \frac{h}{R}(2 + \frac{h}{R})\tan^2\theta\right)^{1/2}\right]}{\pi \sin i}$$

for reasonably high inclinations.

Further, if $\frac{1}{2}\frac{h}{R} << 1$, a binomial expansion would give

$$f = \frac{h \tan\theta}{R\,\pi \sin i} \quad .$$

APPENDIX B

Arbitrariness of the Origin

There will be a periodicity in the pattern of crossing points, the same points being eventually repeated, only if, after a whole number of revolutions, a crossing occurs at the end point of the arc σ. Since the perimeter of a circle of constant latitude has length 1, this condition reduces to

$$n\sigma = m \qquad (n,m \text{ integers}) \tag{B.1}$$

That is, repeated crossings will occur for all rational values of σ.

Conversely, if σ is not rational, the ground track cannot cross any given point more than once. If the process goes on indefinitely the total number of distinct crossing points on σ goes to infinity. Since there is a constant displacement between any pair of successive crossings of the parallel of latitude, any crossing point may be taken as the origin. By symmetry, then, the pattern tends to a situation such that crossing points are equally spaced, and since the total number of crossings becomes infinite on the finite circle, it is obvious that a crossing will eventually occur arbitrarily close to any given point, regardless of the origin.

For this reason, the analysis holds true for any target at a given latitude, as long as σ is an irrational number. For repeated tracks, however, at rational values of σ, there is clearly a basic distinction between points which are crossings and those which are not. If (B.1) holds, there will be n distinct crossing points equally spaced on the unit circle, and by symmetry any one of them may be taken as the origin. No other points lie on a ground track, and the observations of a given target will certainly depend upon its position.

APPENDIX C

The Partition of (σ, f) Space

We prove that the regions of (σ, f) - space in which the maximum number of cycles between sightings (C) is constant, are triangles.

The proof proceeds in several steps.

1. The constant-C regions are polygons - The sightings of a given point occur for those crossings of latitude ϕ included within an arc $\pm \frac{1}{2}f$ from that point. Suppose that for a given configuration there is a certain set of crossings within f. The value of C is determined by only a small number of early crossings. As σ changes the positions of the points change continuously, but as long as they remain within f, there will be no change in C. This situation corresponds to the interior of a constant-C region.

   When f can no longer include one or more of these points, or alternatively, when it includes new points, then C can change value. It is crucial to recognize that changes involve pairs of crossings, since if only one were to move into or out of f, the (arbitrary) position of f could be adjusted to prevent the change. Therefore, changes in C correspond to the case that f is equal to the distance between some pair of crossing points.

   The distance between two crossings which are one cycle apart is

   $$N\sigma - 1$$

   the distance between a pair n cycles apart is

   $$\sum_{i=1}^{n} (N_i \sigma - 1) = \sigma \sum_{i=1}^{n} N_i - n$$

   where N_i may take one of two possible values, as discussed in Section III.

$$f = \left| \sigma \sum_{i=1}^{n} N_i - n \right| \qquad (C.1)$$

is the equation of a straight line in the (σ, f)-plane, therefore, the boundaries of the C-regions are straight lines, and the regions themselves polygons.

2. <u>The vertices of each polygon occur only when crossing points repeat</u> - According to (B.1), crossing points are repeated if $n\sigma = m$. A vertex is a point where two straight line boundaries described by (C.1) intersect. That is

$$f = \left| \sigma \sum_{i=1}^{n_1} N_i - n_1 \right| = \left| \sigma \sum_{j=1}^{n_2} N_j - n_2 \right|$$

n_1, n_2 correspond to different pairs of crossings which, for a vertex, are the same distance apart. But this reduces to

$$\left(\sum^{n_1} N_i \pm \sum^{n_2} N_j \right) \sigma = n_1 \pm n_2$$

which is the same form as (B.1).

Thus, vertices of the constant-C regions occur only at rational values of σ, which correspond to the case of a finite number of different crossing points, each repeated infinitely many times.

3. <u>There is one, and only one vertex for each rational value of σ</u> - A boundary, described by (C.1), is a situation in which the end points of f are crossing points. That (C.1) is a continuous function is a reflection of the fact that with small changes in σ there will be small variations in the positions of these end points. There will also be small changes in the positions of points within f, but as long as they remain within f, C is unchanged.

This continuous behavior breaks down when there are no crossing points within f. Such a situation can occur only with repeated ground tracks, and only when f is sufficiently small that it lies entirely between two adjacent tracks.

Thus, if $\sigma = m/n$ a crossing point is repeated after n revolutions, or m cycles. Then $f < \sigma/m$ describes a region for which $C = \infty$ because the crossing points are spaced $\frac{\sigma}{m}$ apart. When $f = \sigma/m$, an arbitrarily small change in σ will produce infinitely many crossing points within f, so that the point

$$\sigma = \frac{m}{n} , \qquad f = \frac{\sigma}{m} = \frac{1}{n} \tag{C.2}$$

is the junction between infinitely many C-regions. Thus, at each rational value of σ, there is one and only one vertex, given by (C.2).

4. Each C-region is a triangle - Each degenerate region $C = \infty$ terminates at a vertex. For the value $\sigma = m/n$, it must be the lower vertex of a region $C = m$, for if $f > \sigma/m$, there must be at least one crossing point within f, which is repeated every m cycles.

   There can be only one such vertex for each region $C = m$; for, suppose there are several vertices, at $\sigma = m/n_1$, m/n_2, $\cdots$ these must lie on the line $f = \sigma/m$ (C.2). But between any pair there can always be found a point $\sigma = m'/n$, with $m' < m$ and the line $C = \infty$ at that value of σ will intersect the line $f = \sigma/m$, since its end point is at $\sigma/m' > \sigma/m$, giving a second vertex at the same value $\sigma = m'/n$, which is impossible.

   It follows also from this result that each region must be a triangle - any other construction will violate the condition that a line $C=\infty$ may not intersect any boundary except at its terminus point. (A little experimentation will immediately convince the reader of this.)

5. Method of construction of the triangular C-regions The range of σ is determined by the physics of the planet-satellite system. For a low-orbit Earth satellite,

   $$.06 < \sigma < .07$$

   is suitable.

Rational values of σ are $\sigma = m/n$, where m = numbers of cycles, and n = number of revolutions between exact repetitions of a crossing point.

If N_i = number of revolutions in the i'th cycle, then

$$n = \sum_{i=1}^{m} N_i \quad .$$

Recall (Section II) that $N_i = M$ or $M-1$, where $M-1$ = integral part of $\frac{1}{\sigma}$, then $n = mM-r$, $r < m$.

The procedure is

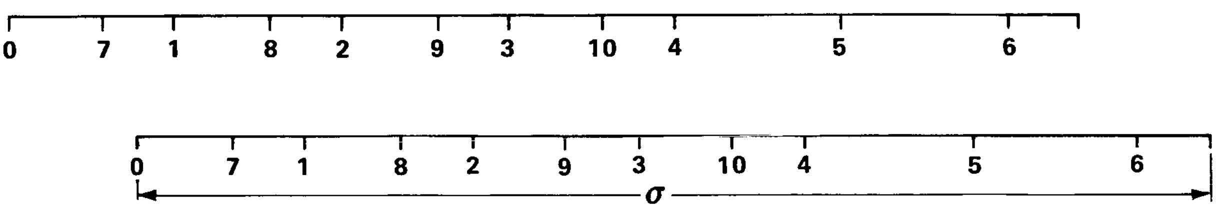
(a) Mark vertices at $\sigma = \frac{m}{mM-r}$, $f = \frac{1}{mM-r}$

(b) Proceed in strictly increasing order $m = 2,3\cdots$ and $r = 1,2\cdots m$ to construct each triangle $C = m$ according to the rule that the lower vertex corresponds to m, the upper vertices to m_1, $m_2 < m$. The construction will be found to be unique.

APPENDIX D

The Complete Sequence of Observations

The analysis has been designed to yield results pertaining to two separate sequences of observations. In order to complete the picture the two sequences must be fitted together. The accompanying diagram shows the beginning of two sequences



of crossings of a given latitude; one for motion of increasing latitude, the other for decreasing latitude. The two are "out of phase" to a degree depending on the time taken for the satellite to pass from, say, latitude Ø, through its point of greatest latitude, and down to Ø.

Suppose this time is τ. According to our previous discussion we need only consider events occurring on an arc σ containing the target. Between two successive crossings of σ at the same latitude, one increasing, the other decreasing in latitude, there will be a whole number Q of periods, plus τ. Consider a crossing of σ in the upward direction. The next downward crossing will be at a distance

$$\left(Q + \frac{\tau}{P}\right)\sigma^{-1} \qquad \text{(P = one period)}$$

from it. Arguing as for N (Section III) we see that in order for this crossing to lie within σ, Q must take one of two values: the integers between which lies the number

$$\left(\frac{1}{\sigma} - \frac{\tau}{P}\right)$$

τ will not remain constant for elliptical orbits but will vary in roughly sinusoidal fashion, being a minimum when perigee occurs at the highest latitude of the orbit, and maximum when apogee occurs there. For circular orbits, however, τ is constant. To evaluate τ, note that the orbit will cross latitude ϕ at longitudes

$$\lambda_{1,2} = \sin^{-1}(\tan \phi / \tan i) \quad .$$

This is the solution of (A.4) with $Z_o=0$.

The corresponding anomalies, measured from the ascending node are

$$\sin^{-1}(\sin \phi / \sin i) \quad .$$

Then, since an entire revolution involves 2π radians,

$$\frac{\tau}{P} = \frac{1}{2\pi} \sin^{-1}(\sin \phi / \sin i) \quad .$$

NOTATION

a	nautical miles	semi-major axis
C	integer	maximum number of cycles between sightings
e		eccentricity
f	pure number	intersection of field-of-view with parallel of latitude
h	n.m.	altitude
i		inclination
J	1.624×10^{-3}	1^{st}-order oblateness parameter of Earth
M		number of revolutions to complete a circumnavigation of the globe.
N		number of revolutions between successive crossings of an arc σ (= one cycle)
P		period of revolution
R	3444 n.m.	radius of Earth
γ		angle of Sun in ecliptic
ε	23°27'	obliquity of ecliptic
θ		half-angle of view of observing instrument
μ	$(2.1643 \times 10^7)^2$ n.m.^{3}/day^2	geo-gravitational mass constant (based on mean solar day)
ϕ		latitude
σ		arc of constant latitude between successive ground tracks
X		angle in ecliptic between orbital plane and sun-line
Ω		longitude of ascending mode